Separability and exchangeability of many-qubit states

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A pure or mixed state of N qubits (or spin-1/2 particles) is *separable* if it can be written as a classical mixture of product states. Separable states are not entangled, and they can be understood in purely classical terms: the spins can be thought of as pointing, with probability p_k, in a set of directions in space labeled k, where $k = 1, \ldots, N$. Nevertheless, separable states can be useful for quantum computing. For instance, a version of Grover's search algorithm has been found that needs no entanglement [1].

Any mixed state can be written as a classical mixture in a large number of ways. The states arising in NMR quantum computing are usually written in the form of *pseudopure* states, but there also exists decompositions into product states, showing that up to now, NMR experiments have not generated entangled states [2]. This fact has led to a controversy on whether NMR experiments have demonstrated genuine quantum information processing. We will discuss the extent to which readout measurements and quantum gate operations in NMR can be modelled classically [3].

Exchangeability is a much less known property of many-qubit states. By extension of the classical concept due to de Finetti [4], an N-qubit state, pure or mixed, is exchangeable if it is invariant under permutations of the qubits, and if for any $M > 0$, there is a state of $N + M$ qubits that is invariant under permutations of the qubits, such that the N-qubit state can be obtained by a trace over the additional M qubits. Exchangeability implies separability. For exchangeable states, the problem of quantum inference, or quantum state estimation, becomes very simple, as in this case one can derive a quantum equivalent of Bayes's rule. We present a simple derivation of the rule from the theory of generalized measurements and give a simple example.

[1] S. Lloyd, e-print quant-ph/9903057.

[2] S. L. Braunstein, C. M. Caves, R. Jozsa, N. Linden, S. Popescu, and R. Schack, Phys. Rev. Lett. **83**, 1054 (1999).

[3] R. Schack and C. M. Caves, Phys. Rev. A **60**, 4354 (1999).

[4] B. de Finetti, Theory of Probability (Wiley, New York, 1990).